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The impact of tidal errors on the determination of the Lense-Thirring effect from satellite laser ranging

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Abstract

The general relativistic Lense-Thirring effect can be detected by means of a suitable combination of orbital residuals of the laser-ranged LAGEOS and LAGEOS II satellites. While this observable is not affected by the orbital perturbation induced by the zonal Earth solid and ocean tides, it is sensitive to those generated by the tesseral and sectorial tides. The assessment of their influence on the measurement of the parameter μ_{LT}, with which the gravitomagnetic effect is accounted for, is the goal of this paper. After simulating the combined residual curve by calculating accurately the mismodeling of the more effective tidal perturbations, it has been found that, while the solid tides affect the recovery of μ_{LT} at a level always well below 1%, for the ocean tides and the other long-period signals $\Delta\mu$ depends strongly on the observational period and the noise level: $\Delta\mu_{tides} \simeq 2\%$ after 7 years. The aliasing effect of K_1 $l=3$ $p=1$ tide and SRP(4241) solar radiation pressure harmonic, with periods longer than 4 years, on the perigee of LAGEOS II yield to a maximum systematic uncertainty on μ_{LT} of less than 4% over different observational periods. The zonal 18.6-year tide does not affect the combined residuals.

1 Introduction

According to Ciufolini (1996), the tiny general relativistic gravitomagnetic effect (Lense & Thirring 1918) can be detected by analyzing the orbits of the two laser-ranged LAGEOS and LAGEOS II satellites. The basic equation is (Ciufolini 1996):

$$\dot{y} \equiv \delta\dot{\Omega}^{I}_{exp} + k_1 \times \delta\dot{\Omega}^{II}_{exp} + k_2 \times \delta\dot{\omega}^{II}_{exp} = \mu \times 60.2. \tag{1}$$

In it $k_1 = 0.295$, $k_2 = -0.35$, μ is the scaling parameter, equal to 1 in general relativity and 0 in classical mechanics, to be determined and $\delta\Omega^{I}_{exp}$, $\delta\Omega^{II}_{exp}$, $\delta\omega^{II}_{exp}$ are the orbital residuals, in milliarcseconds (mas), calculated with the aid of orbit determination software like UTOPIA [Center for Space Research, Univ. of Texas at Austin] or GEODYN [NASA Goddard Space Flight Center], of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II. The residuals account for any unmodeled or mismodeled physical phenomena acting on the observable analyzed. By treating the gravitomagnetism as an unmodeled physical effect, general relativity forecasts its effect on eq.(1) to be:

$$\dot{y}_{LT} \equiv (31\ mas/y) + k_1 \times (31.5\ mas/y) + k_2 \times (-57\ mas/y) \simeq 60.2\ mas/y. \tag{2}$$

The determination of μ is influenced by a great number of gravitational and nongravitational perturbations acting upon LAGEOS and LAGEOS II. Among the perturbations of gravitational origin a primary role is played by Earth's solid and ocean tides.

The combined residuals $y \equiv \delta\Omega^{I}_{exp} + k_1\delta\Omega^{II}_{exp} + k_2\delta\omega^{II}_{exp}$ allow to cancel out the static and dynamical contributions of degree $l = 2, 4$ and order $m = 0$ of the geopotential. However, this is not so for the tesseral ($m = 1$) and sectorial ($m = 2$) tides.

This paper aims to assess quantitatively, in a reliable and plausible manner, how the solid and ocean Earth tides of order $m = 1, 2$ affect the recovery of μ by simulating the real residual curve and analyzing it. The T_{obs} chosen span ranges from 4 years to 8 years: this is so because 4 years is the length of the latest time series actually analysed by Ciufolini et al. (1998) and 8 years is the maximum length obtainable today because LAGEOS II was launched in 1992. The analysis includes also the long-period signals due to solar radiation pressure and the $l = 3,\ m = 0$ geopotential harmonic acting on the perigee of LAGEOS II. In our case we have a signal built up with a secular trend, i.e. the Lense-Thirring effect, and a certain number of

long-period harmonics, i.e. the tesseral and sectorial tidal perturbations and the other signals with known periodicities. The part of interest for us is the secular trend while the harmonic part represents the noise. We address the problem of how the harmonics affect the recovery of the secular trend on given time spans T_{obs} and various samplings Δt.

Among the long-period signals we distinguish between those signals whose periods are shorter than T_{obs} and those with periods longer than T_{obs}. While the former average out if T_{obs} is an integer multiple of their periods, the action of the latter is more subtle since they could resemble a trend over temporal intervals too short with respect to their periods. They must be considered therefore as biases on the Lense-Thirring determination affecting its recovery by means of their mismodeling. Thus, it is of the utmost importance that we reliably assess their impact on the determination of the trend from the gravitomagnetic effect. It would be useful to direct the efforts of the community (geophysicists, astronomers and space geodesists) towards the improvement of our knowledge of those tidal constituents to which μ turns out to be particularly sensitive (for the LAGEOS' orbits).

This investigation will quantify unambiguously what one means with statements like: $\Delta\mu_{tides} \leq X\%\mu_{LT}$. In this paper we shall try to put forward a simple and meaningful approach. It must be pointed out though that it is not a straightforward application of any exact or rigorously proven method; on the contrary, it is, at a certain level, heuristic and intuitive, but it has the merit of yielding reasonable and simple answers and allowing for their critical discussion.

The paper is organized as follows. In Section 2 the mismodeling of the various tidal orbit perturbations is worked out in order to use these estimates in the simulation procedure of the synthetic observable curve whose features are outlined in Section 3. In Section 4 the effects of the harmonics with period shorter than 5 years are examined by comparing the least squares fitted values of μ in two different scenarios: in the first one the simulated curve is complete and the mathematical model we fit contains all the most relevant signals plus a straight line. In the second one the harmonics are removed from the simulated curve which is fitted by means of the straight line only. Section 5 address the topic of those harmonics with periods longer than 5 years and Section 6 is devoted to the conclusions.

2 The mismodeling of the tidal perturbations

Since the observable is a residual curve, if we want to simulate it we need reliable estimates of the commission errors of the various tides considered. They will take the place of real residuals in the simulated data. In this section we address this topic by calculating the mismodeled amplitudes of the solid and ocean tidal perturbations on the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II and comparing them to the corresponding gravitomagnetic perturbations over 4 years. A cutoff of 1% has been set in order to obtain a preliminary evaluation of the importance of the various constituents.

Concerning the solid Earth tides, their perturbations on the node and the perigee of an artificial satellite, for a given frequency f, are given by:

$$\Delta\Omega_f = \frac{g}{na^2\sqrt{1-e^2}\sin i}\sum_{l=0}^{\infty}\sum_{m=0}^{l}(\frac{R}{r})^{l+1}\times$$

$$\times A_{lm}\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}\frac{dF_{lmp}}{di}G_{lpq}\frac{1}{f_p}k_{lm}^{(0)}(f)H_l^m\sin\gamma_{flmpq}, \tag{3}$$

$$\Delta\omega_f = \frac{g}{na^2\sqrt{1-e^2}}\sum_{l=0}^{\infty}\sum_{m=0}^{l}(\frac{R}{r})^{l+1}\times$$

$$\times A_{lm}\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}[\frac{1-e^2}{e}F_{lmp}\frac{dG_{lpq}}{de}-\frac{\cos i}{\sin i}\frac{dF_{lmp}}{di}G_{lpq}]\frac{1}{f_p}k_{lm}^{(0)}(f)H_l^m\sin\gamma_{flmpq}, \tag{4}$$

while for the ocean tides we have:

$$\Delta\Omega_f = \frac{1}{na^2\sqrt{1-e^2}\sin i}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\sum_{+}^{-}(\frac{R}{r})^{l+1}A_{lmf}^{\pm}\times$$

$$\times\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}\frac{dF_{lmp}}{di}G_{lpq}\frac{1}{f_p}\begin{bmatrix}\sin\gamma_{flmpq}^{\pm}\\ -\cos\gamma_{flmpq}^{\pm}\end{bmatrix}_{l-m\ odd}^{l-m\ even}, \tag{5}$$

$$\Delta\omega_f = \frac{1}{na^2\sqrt{1-e^2}}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\sum_{+}^{-}(\frac{R}{r})^{l+1}A_{lmf}^{\pm}\times$$

$$\times\sum_{p=0}^{l}\sum_{q=-\infty}^{+\infty}[\frac{1-e^2}{e}F_{lmp}\frac{dG_{lpq}}{de}-\frac{\cos i}{\sin i}\frac{dF_{lmp}}{di}G_{lpq}]\frac{1}{f_p}\begin{bmatrix}\sin\gamma_{flmpq}^{\pm}\\ -\cos\gamma_{flmpq}^{\pm}\end{bmatrix}_{l-m\ odd}^{l-m\ even}. \tag{6}$$

The main source of uncertainties in such expressions are the $l=2$ Love numbers $k_{lm}^{(0)}(f)$, the load Love numbers k_l', the coefficients C_{lmf}^{+} and the orbital injection errors δi affecting the inclination i.

As far as the solid tidal constituents, by assuming $\delta i = 0.5$ mas (Ciufolini 1989), we have calculated $\left| \frac{\partial A(\Omega)}{\partial i} \right| \delta i$ and $\left| \frac{\partial A(\omega)}{\partial i} \right| \delta i$ for 055.565, K_1, and S_2 which are the most powerful tidal constituents in perturbing LAGEOS and LAGEOS II orbits. The results are of the order of 10^{-6} mas, so that we can neglect the effect of uncertainties in the inclination determination. Concerning the Love numbers $k_{2m}^{(0)}(f)$, we have assessed the uncertainties on them by calculating for certain tidal constituents the factor $\delta k_2/k_2$; k_2 is the average on the values released by the most reliable models and δk_2 is its standard deviation. According to the recommendations of the Working Group of Theoretical Tidal Model of the Earth Tide Commission (http://www.astro.oma.be/D1/EARTH_TIDES/wgtide.html), in the diurnal band we have chosen the values released by Mathews et al. (1995), McCarthy (1996) and the two sets by Dehant et al. (1999). For the zonal and sectorial bands we have included also the results of Wang (1994). The uncertainties calculated in the Love numbers k_2 span from 0.5% to 1.5% for the tides of interest. However, it must be noted that the worst known Love numbers are those related to the zonal band of the tidal spectrum due to the uncertainties on the anelasticity of the Earth' s mantle. These results were obtained in order to calculate the mismodeled amplitudes of the solid tidal perturbations $\delta\Omega^{I}$, $\delta\Omega^{II}$ and $\delta\omega^{II}$; they were subsequently compared with the gravitomagnetic precessions over 4 years $\Delta\Omega^{I}_{LT} = 124$ mas, $\Delta\Omega^{II}_{LT} = 126$ mas and $\Delta\omega^{II}_{LT} = -228$ mas. In Tab.1 we have quoted only those tidal lines whose mismodeled perturbative amplitudes are greater than 1% of the gravitomagnetic perturbations. It turns out that only 055.565 18.6-year and K_1 exceed this cutoff. Remember that the 18.6 year tide cancels out, so that we can conclude that the uncertainties on the Love numbers affect the combined residuals at a level $\leq 1\%$.

Concerning the ocean loading, the first calculations of the load Love numbers $k_l^{'}$ are due to Farrell (1972). Pagiatakis (1990) in a first step has recalculated $k_l^{'}$ for an elastic, isotropic and non-rotating Earth: for $l < 800$ he claims that his estimates differ from those by Farrell (1972), calculated with the same hypotheses, by less than 1%. Subsequently, he added to the equations, one at a time, the effects of anisotropy, rotation and dissipation; for low values of l, their effect on the results of the calculations amount to less than 1%. For the ocean loading coefficients see also (Scherneck, 1991). It has been decided to calculate $\left| \frac{\partial A(\omega)}{\partial k_l^{'}} \right| \delta k_l^{'}$ for the perigee of LAGEOS II for K_1 $l = 3$ $p = 1$ which turns out to be the most powerful

ocean tidal constituent acting on this orbital element. First, we have calculated mean and standard deviation of the values for $k_3^{'}$ by Farrell and Pagiatakis obtaining $\delta k_3^{'}/k_3^{'} = 0.9\%$, in agreement with the estimates by Pagiatakis. Then, assuming pessimistically that the global effect of the departures from these symmetric models results in a total $\delta k_3^{'}/k_3^{'} = 2\%$, we have obtained $\delta\omega^{II} = 5.5$ mas which corresponds to 2% of $\Delta\omega_{LT}^{II}$ over 4 years. Subsequently, for this constituent and for all other tidal lines we have calculated the effect of the mismodeling of C_{lmf}^{+} as quoted in EGM96 (Lemoine et al. 1998). In Tab.2 we compare the so obtained mismodeled ocean tidal perturbations to those generated over 4 years by the Lense-Thirring effect. It turns out that the perigee of LAGEOS II is more sensitive to the mismodeling of the ocean part of the Earth response to the tide generating potential. In particular, the effect of K_1 $l = 3$ $p = 1$ $q = -1$ is relevant with a total $\delta\omega = \left|\frac{\partial A(\omega)}{\partial C_{lmf}^{+}}\right| \delta C_{lmf}^{+} + \left|\frac{\partial A(\omega)}{\partial k_l^{'}}\right| \delta k_l^{'}$ of 64.5 mas amounting to 28.3 % of $\Delta\omega_{LT}$ over 4 years.

Mismodeled solid tidal perturbations on nodes of LAGEOS and LAGEOS II and perigee of LAGEOS II							
Tide	$\Delta\Omega_{LT}^{I} = 124$ mas $\Delta\Omega_{LT}^{II} = 126$ mas $\Delta\omega_{LT}^{II} = -228$ mas T_{obs}=4 years						
	$\delta k_2/k_2$ (%)	$\delta\Omega^{I}$ (mas)	$\delta\Omega^{I}/\Delta\Omega_{LT}^{I}$ (%)	$\delta\Omega^{II}$ (mas)	$\delta\Omega^{II}/\Delta\Omega_{LT}^{II}$ (%)	$\delta\omega^{II}$ (mas)	$\delta\omega^{II}/\Delta\omega_{LT}^{II}$ (%)
055.565	1.5	-16.5	13.3	30.3	24	-21	9.2
165.555 K_1	0.5	9	7.2	-2	1.6	10.2	4.4

Table 1: Mismodeled solid tidal perturbations on nodes Ω of LAGEOS and LAGEOS II and the perigee ω of LAGEOS II compared to their gravitomagnetic precessions over 4 years. The percent variation refers to the ratios of the mismodeled amplitudes of the tidal harmonics to the gravitomagnetic perturbations over 4 years.

Mismodeled ocean tidal perturbations on nodes of LAGEOS and LAGEOS II and perigee of LAGEOS II							
Tide	$\Delta\Omega_{LT}^{I} = 124$ mas $\Delta\Omega_{LT}^{II} = 126$ mas $\Delta\omega_{LT}^{II} = -228$ mas $T_{obs} = 4$ years						
	$\delta C^{+}/C^{+}$ (%)	$\delta\Omega^{I}$ (mas)	$\delta\Omega^{I}/\Delta\Omega_{LT}^{I}$ (%)	$\delta\Omega^{II}$ (mas)	$\delta\Omega^{II}/\Delta\Omega_{LT}^{II}$ (%)	$\delta\omega^{II}$ (mas)	$\delta\omega^{II}/\Delta\omega_{LT}^{II}$ (%)
S_a l=2 p=1 q=0	6.7	1.37	1.1	2.5	1.9	-	-
S_a l=3 p=1 q=-1	10	-	-	-	-	11.4	5
S_a l=3 p=2 q=1	10	-	-	-	-	29.7	13
S_{sa} l=3 p=1 q=-1	27.2	-	-	-	-	6.2	2.7
S_{sa} l=3 p=2 q=1	27.2	-	-	-	-	9.8	4.3
K_1 l=2 p=1 q=0	3.8	5.9	4.7	1.3	1	6.75	2.9
K_1 l=3 p=1 q=-1	5.2	-	-	-	-	64.5	28.3
K_1 l=3 p=2 q=1	5.2	-	-	-	-	18	7.9
K_1 l=4 p=2 q=0	3.9	-	-	1.6	1.2	-	-
P_1 l=3 p=1 q=-1	18.5	-	-	-	-	5.3	2.3
P_1 l=3 p=2 q=1	18.5	-	-	-	-	6.4	2.8
K_2 l=3 p=1 q=-1	5.5	-	-	-	-	11.7	5
K_2 l=3 p=2 q=1	5.5	-	-	-	-	4.8	2
S_2 l=3 p=1 q=-1	7.1	-	-	-	-	6.9	3
S_2 l=3 p=2 q=1	7.1	-	-	-	-	4.4	1.9
T_2 l=3 p=1 q=-1	50	-	-	-	-	2.6	1.1

Table 2: Mismodeled ocean tidal perturbations on nodes Ω of LAGEOS and LAGEOS II and the perigee ω of LAGEOS II compared to their gravitomagnetic precessions over 4 years. The effect of the ocean loading has been neglected. When the 1% cutoff has not been reached a - has been inserted. The values quoted for K_1 $l = 3$ $p = 1$ includes also the mismodeling in the ocean loading coefficient $k_3^{'}$ assumed equal to 2%. The percent variation refers to the ratios of the mismodeled amplitudes of the tidal harmonics to the gravitomagnetic perturbations over 4 years.

3 The simulated residual signal

The first step of our strategy was to generate with MATLAB a time series which simulates, at the same length and resolution, the real residual curve obtainable from $y = \delta\Omega^{I}_{exp} + k_1\delta\Omega^{II}_{exp} + k_2\delta\omega^{II}_{exp}$ through, e.g., GEODYN. This simulated curve ("Input Model" - IM in the following) was constructed with:

• The secular Lense-Thirring trend as predicted by the general relativity[1]

• A certain number of sinusoids of the form $\delta A_k \cos\left(\frac{2\pi}{P_k}t + \phi_k\right)$ with known periods $P_k,\ k = 1,..N$ simulating the mismodeled tides and other long-period signals which, to the level of assumed mismodeling, affect the combined residuals

• A noise of given amplitude which simulates the experimental errors on the laser-ranged measurements and, depending upon the characteristics chosen for it, some other physical forces.

In a nutshell:

$$IM = LT + [mismodeled\ tides] + [other\ mismodeled\ long\ period\ signals] + [noise]. \quad (7)$$

The harmonics included in the IM are the following:

• $K_1,\ l = 2$ solid and ocean; node of LAGEOS (P=1043.67 days)

• $K_1,\ l = 2$ solid and ocean; node and perigee of LAGEOS II (P=-569.21 days)

• $K_1,\ l = 3,\ p = 1$ ocean; perigee of LAGEOS II (P=-1851.9 days)

• $K_1,\ l = 3,\ p = 2$ ocean; perigee of LAGEOS II (P=-336.28 days)

• $K_2,\ l = 3,\ p = 1$ ocean; perigee of LAGEOS II (P=-435.3 days)

• $K_2,\ l = 3,\ p = 2$ ocean; perigee of LAGEOS II (P=-211.4 days)

• 165.565, $l = 2$ solid; node of LAGEOS (P=904.77 days)

• 165.565, $l = 2$ solid: node and perigee of LAGEOS II (P=-621.22 days)

• $S_2,\ l = 2$ solid and ocean; node of LAGEOS (P=-280.93 days)

• $S_2,\ l = 2$ solid and ocean; node and perigee of LAGEOS II (P=-111.24 days)

• $S_2,\ l = 3,\ p = 1$ ocean; perigee of LAGEOS II (P=-128.6 days)

• $S_2,\ l = 3,\ p = 2$ ocean; perigee of LAGEOS II (P=-97.9 days)

• $P_1,\ l = 2$ solid and ocean; node of LAGEOS (P=-221.35 days)

[1]Remember that in the dynamical models of GEODYN II it was set purposely equal to 0 so that the residuals absorbed (contained) entirely the relativistic effect (Ciufolini et al. 1997).

- $P_1,\ l=2$ solid and ocean; node and perigee of LAGEOS II (P=-138.26 days)
- $P_1,\ l=3,\ p=1$ ocean; perigee of LAGEOS II (P=-166.2 days)
- $P_1,\ l=3,\ p=2$ ocean; perigee of LAGEOS II (P=-118.35 days)
- *Solar Radiation Pressure*, perigee of LAGEOS II (P=-4241 days)
- *Solar Radiation Pressure*, perigee of LAGEOS II (P=657 days)
- *perigee odd zonal* C_{30}, perigee of LAGEOS II (P=821.79)

In the following the signals due to solar radiation pressure will be denoted as SRP(P) where P is their period; the effects of the eclipses and Earth penumbra have not been accounted for. Many of the periodicities listed above have been actually found in the Fourier spectrum of the real residual curve (Ciufolini et al. 1997). Concerning K_1 $l=3$ $p=1$ and SRP(4241), see Section 5.

When the real data are collected they refer to a unique, unrepeatable situation characterized by certain starting and ending dates for T_{obs}. This means that each analysis which could be carried out in the real world necessarily refers to a given set of initial phases and noise for the residual curve corresponding to the chosen observational period; if the data are collected over the same T_{obs} shifted backward or forward in time, in general, such features of the curve will change. We neither know a priori when the next real experiment will be performed, nor which will be the set of initial phases and the level of experimental errors accounted for by the noise. Moreover, maybe the dynamical models of the orbit determination software employed are out of date with regard to the perturbations acting upon satellites' orbits or do not include some of them at all. Consequently, it would be incorrect to work with a single simulated curve, fixed by an arbitrary set of $\delta A_k,\ \phi_k$ and noise, because it could refer to a situation different from that in which, in the real world, the residuals will actually correspond.

The need for great flexibility in generating the IM becomes apparent: to account for the entire spectrum of possibilities occurring when the real analyses will be carried out. We therefore decided to build into the MATLAB routine the capability to vary randomly the initial phases ϕ_k, the noise and the amplitude errors δA_k. Concerning the error amplitudes of the harmonics, they can be randomly varied so that $\delta A_k \in [0,\ \delta A_k^{nom}]$ where δA_k^{nom} is the nominal mismodeled amplitude calculated taking into account Tab.1 and Tab.2; it means that we assume they are reliable estimates of the differences between the real data and the dynamical models of the

orbital determination softwares, i.e. of the residuals. The MATLAB routine allows also the user to decide which harmonic is to be included in the IM; it is also possible to choose the length of the time series T_{obs}, the sampling step Δt and the amplitude of the noise. Fig.(1) shows a typical simulation over 3.1 years with Δt= 15 days and a given set of random initial phases and noise: all the long period signals have been included with $\delta A_k = \delta A_k^{nom}$. It can be compared to the real residual curve released in (Ciufolini et al. 1997) for the same time step and time span going from November 1992 to December 1995: qualitatively they agree very well. We also calculated the root mean square for the IM simulated data, of 9 mas.

In order to assess quantitatively this feature we proceeded as follows. First, over a time span of 3.1 years, we fitted the IM with a straight line only, finding for a choice of random phases and noise which qualitatively reproduces the curve shown in (Ciufolini et al. 1997), the value of 38.25 mas for the root mean square of the post fit IM. The value quoted in (Ciufolini et al. 1997) is 43 mas. Secondly, as done in the cited paper, we fitted the complete IM with the LT plus a set of long-period signals (see Section 4) and then we subtracted the so adjusted harmonics from the original IM obtaining a "residual" simulated signal curve. The latter was subsequently fitted with a straight line only, finding a rms post fit of 12.3 mas (it is nearly independent of the random phases and the noise) versus 13 mas quoted in (Ciufolini et al. 1997). A uniformly distributed noise with nonzero average and amplitude of 50 mas was used (see Section 4). These considerations suggest that the simulation procedure adopted is reliable, replicates the real world satisfactorily and yields a good starting point for conducting our sensitivity analyses.

4 Sensitivity analysis

A preliminary analysis was carried out in order to evaluate the importance of the mismodeled solid tides on the recovery of μ. We calculated the average:

$$\frac{1}{T_{obs}}\int_0^{T_{obs}}[solid\ tides]dt, \tag{8}$$

where $[solid\ tides]$ denotes the analytical expressions of the mismodeled solid tidal perturbations as given by eqs.(3)-(4). Subsequently, we compared it to the value of the gravitomagnetic trend for the same T_{obs}. The analyses were repeated by varying Δt, T_{obs} and the initial phases. They

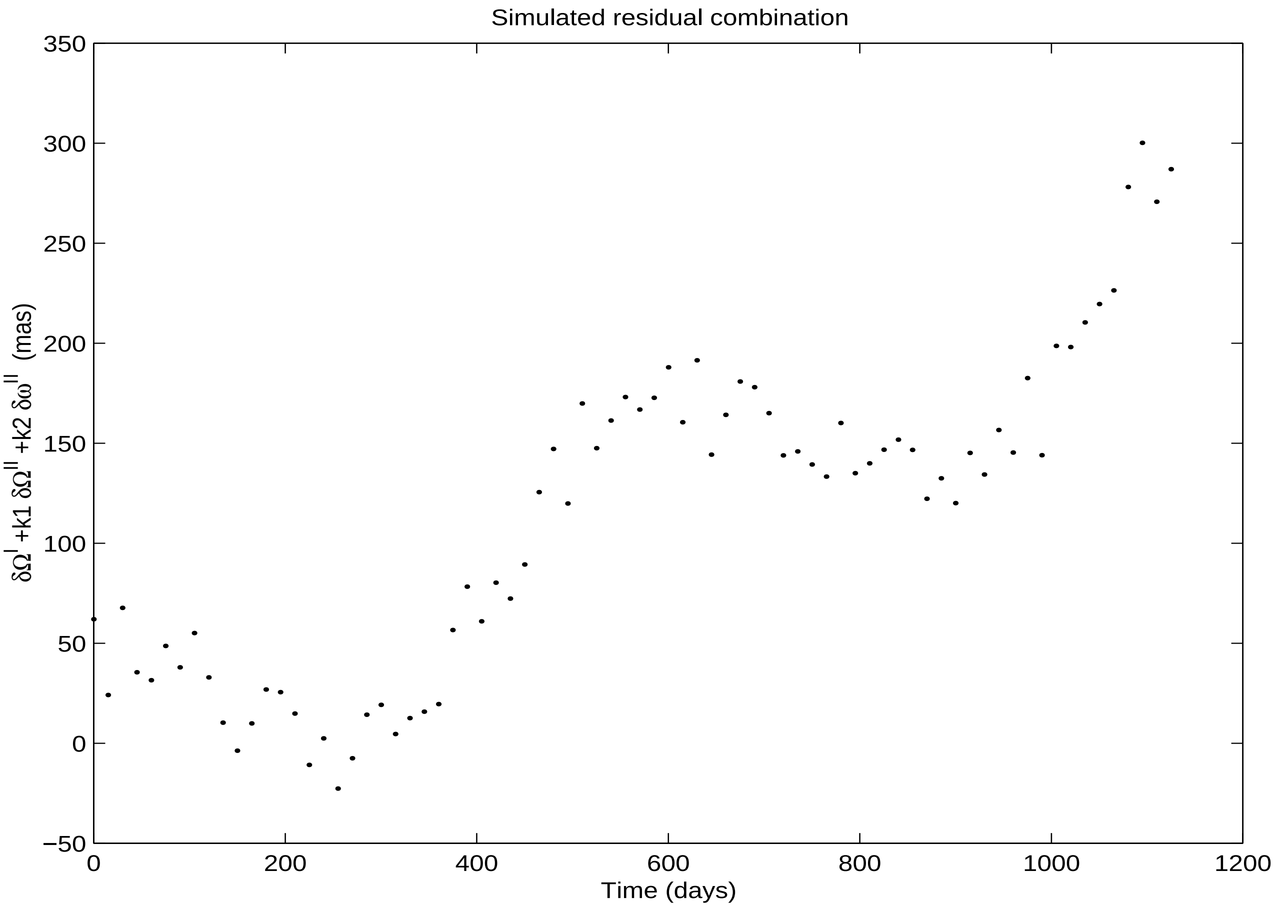

Figure 1: Simulated residual curve. The time span is 3.1 years, the time step is 15 days and all the long period signals are included. The random initial phases and the noise have been chosen in order to reproduce as closely as possible the residual curve of (Ciufolini et al. 1997). The rms of the simulated data amounts to almost 9 mas, the 5% of the preticted gravitomagnetic value for the combined residuals.

have shown that the mismodeled part of the solid Earth tidal spectrum is entirely negligible with respect to the LT signal, falling always below 1% of the gravitomagnetic shift of the combined residuals accumulated over the examined T_{obs}. For the ocean tides and the other long-period signals the problem was addressed in a different way. First, for a given Δt and different time series lengths, we included in the IM the effects of LT, the noise and the solid tides only: subsequently we fitted it simply by means of a straight line. In a second step we have simultaneously added, both to the IM and the fitting model (FM hereafter), all the ocean tides, the solar radiation pressure SRP(657) signal and the odd zonal geopotential harmonic. We then compared the fitted values of μ and $\delta\mu$ recovered from both cases in order to evaluate $\Delta\mu$

and $\Delta\delta\mu$. The least squares fits (Bard 1974; Draper & Smith 1981) were performed by means of the MATLAB routine "nlleasqr" (see, e.g., http://www.ill.fr/tas/matlab/doc/mfit.html); as $\delta\mu$ we have assumed the square root of the diagonal covariance matrix element relative to the slope parameter. Note that the SRP(4241) has never been included in the FM (See Section 5), while K_1 $l = 3$ $p = 1$ has been included for $T_{obs} > 5$ years. For both of the described scenarios we have taken the average for μ and $\delta\mu$ over 1500 runs performed by varying randomly the initial phases, the noise and the amplitudes of the mismodeled signals in order to account for all possible situations occurring in the real world, as pointed out in the previous section. The large number of repetitions was chosen in order to avoid that statistical fluctuations in the results could "leak" into $\Delta\overline{\mu}$ and $\Delta\overline{\delta\mu}$ and corrupt them at the level of 1%. With 1500 runs the standard deviations on $\overline{\mu}$ and $\overline{\delta\mu}$ are of the order of 10^{-3} or less, so that we can reliably use the results of such averages for our comparisons of $\overline{\mu}$(no tides) vs $\overline{\mu}$(all tides).

Before implementing such strategy for different T_{obs}, Δt and noise we carefully analyzed the $\Delta t = 15$ days, T_{obs}=4 years experiment considered in (Ciufolini et al. 1998) by trying to obtain the quantitative features outlined there, so that we start from a firm and reliable basis. This goal was achieved by proceeding as outlined in the previous Section and adopting a uniformly distributed random noise with an amplitude of 35 mas.

Fig.2 shows the results for $\Delta\overline{\mu}$ obtained with $\Delta t = 15$ days and a uniform random noise with amplitudes of 50 mas and 35 mas corresponding to the characteristics of the real curves in (Ciufolini et al. 1997; 1998). Note that our estimates for the case $T_{obs} = 4$ years almost coincide with those by Ciufolini et al. (1998) who claim $\Delta\mu_{tides} \leq 4\%$. Note that for $T_{obs} > 7$ years the effect of tidal perturbations errors falls around 2%. By choosing different Δt does not introduce appreciable modifications to the results presented here. This fact was tested by repeating the set of runs with $\Delta t = 7$ and 22 days.

Up to this point we dealt with the entire set of long-period signals affecting the combined residuals; now we ask if it is possible to assess individually the influence of each tide on the recovery of LT. We shall focus our attention on the case $\Delta t = 15$ days, T_{obs}=4 years.

In order to evaluate the influence of each tidal constituent on the recovery of μ two complementary approaches could be followed in principle. The first one consists in starting without any long-period signal either in the IM or in the FM, and subsequently adding to them one

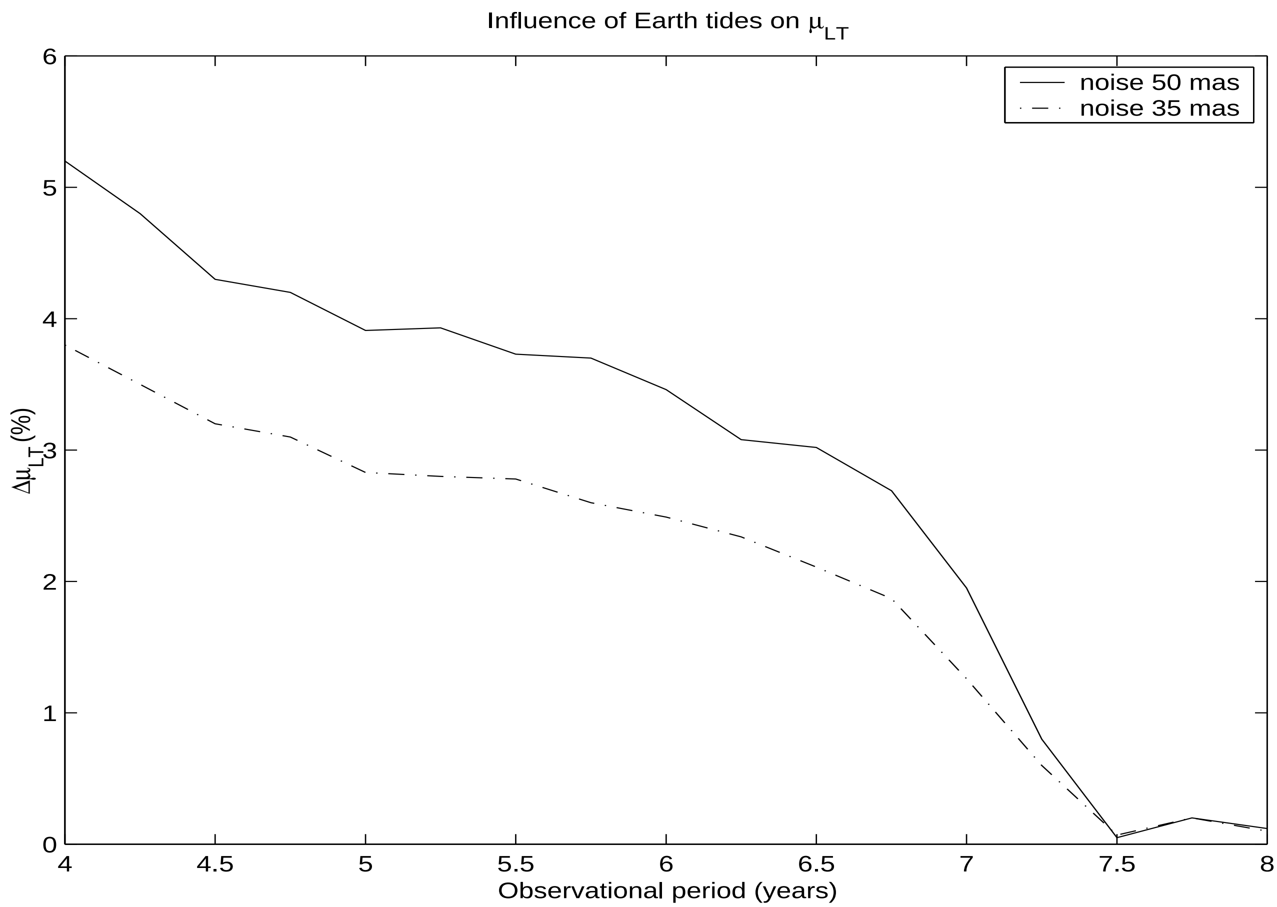

Figure 2: Effects of the long period signals on the recovery of μ_{LT} for Δt=15 days and different choice of uniform random noise. Each point in the curves represents an average over 1500 runs performed by varying randomly the initial phases and the noise' s pattern. $\Delta\mu$ is the difference between the least squares fitted value of μ when both the IM and the FM includes the LT plus all the harmonics and that obtained without any harmonic both in the IM and in the FM.

harmonic at a time, while neglecting all the others. In doing so it is implicitly assumed that each constituent is mutually uncorrelated with any other one present in the signal. In fact, the matter is quite different since if for the complete model case we consider the covariance and correlation matrices of the FM adjusted parameters it turns out that the LT is strongly correlated, at a level of $\mid Corr(i,j)\mid > 0.9$, with certain harmonics, which happen to be mutually correlated too. These are:

- $K_1,\ l=2$ (P=1043.67 days; 1.39 cycles completed)
- $K_1,\ l=2$ (P=-569.21 days; 2.56 cycles completed)
- $K_1,\ l=3,\ p=2$ (P=-336.28 days; 4.34 cycles completed)

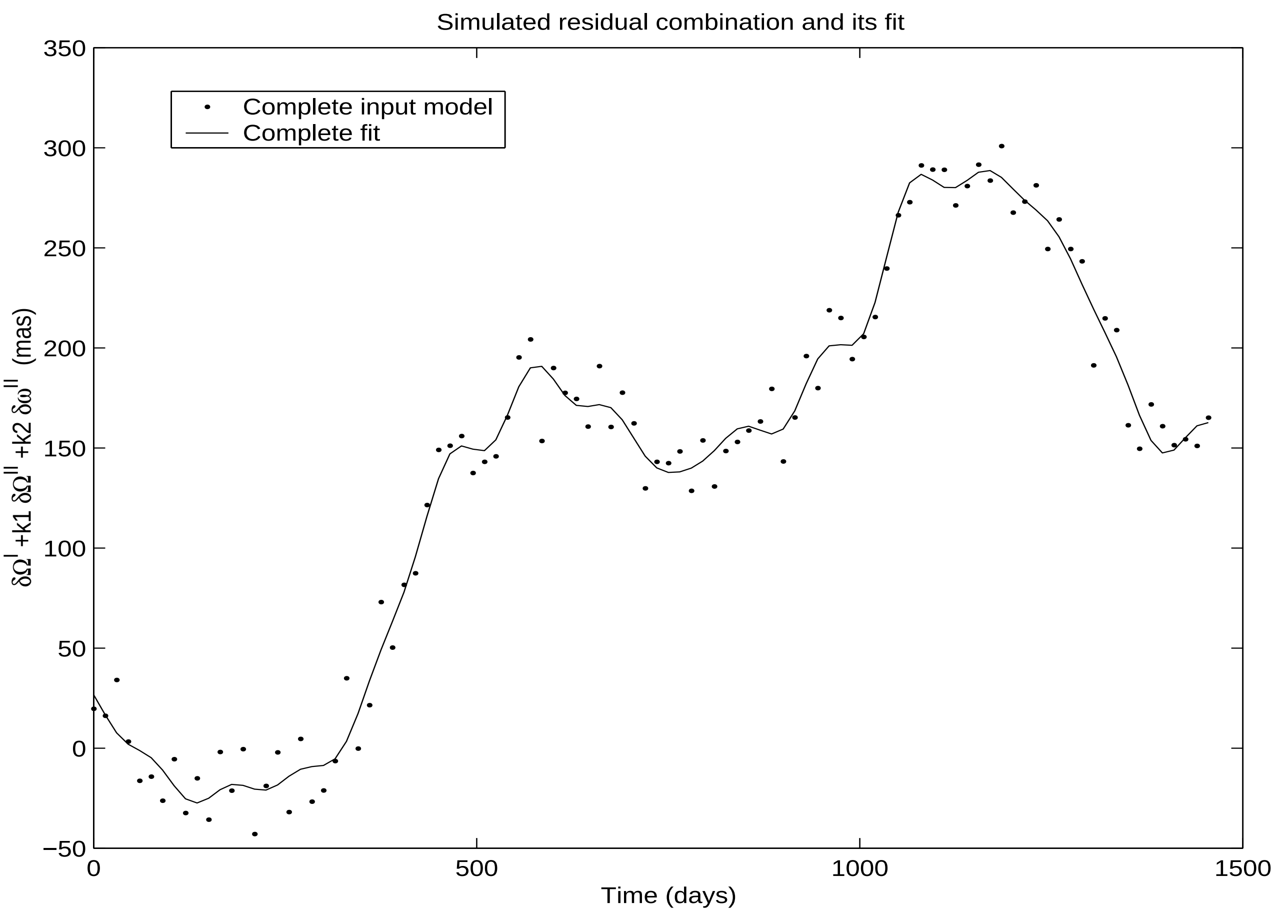

Figure 3: Simulated residual curve and related fit. The time span is 4 years, the time step is 15 days, the simulated data RMS amounts to 9 mas and all the long period signals are included in the simulated data. The initial phases and the noise are random.

- $K_2,\ l=3,\ p=1$ (P=-435.3 days; 3.35 cycles completed)
- *Solar Radiation Pressure*, (P=657 days; 2.22 cycles completed)
- *perigee odd zonal* C_{30}, (P=821.79; 1.77 cycles completed)

Their FM parameters are indeterminate in the sense that the values estimated are smaller than the relative uncertainties assumed to be $\sqrt{Cov(i,i)}$. On the contrary, there are other signals which are poorly correlated to the LT and whose reciprocal correlation too is very low and that are well determined. These are:

- $K_2,\ l=3,\ p=2$ (P=-211.4 days; 6.91 cycles completed)
- $S_2,\ l=3,\ p=1$ (P=-128.6 days; 11.3 cycles completed)
- $S_2,\ l=3,\ p=2$ (P=-97.9 days; 14.9 cycles completed)

- $P_1,\ l=3,\ p=1$ (P=-166.2 days; 8.78 cycles completed)
- $P_1,\ l=3,\ p=2$ (P=-118.35 days; 12.3 cycles completed)

In Fig.(3) the complete IM and FM are shown for a given choice of the initial phases, uniform noise with amplitude of 50 mas, $T_{obs}=4$ years and $\Delta t=$ days. It is interesting to note that over $T_{obs}=4$ years the signals uncorrelated with the LT have in general described many complete cycles, contrary to these correlated with LT. This means that the signals that average out over T_{obs} decorrelate with LT, allowing the gravitomagnetic trend to emerge clearly against the background, almost not affecting the LT recovery. This feature has been tested as follows. In a first step, all the uncorrelated tides have been removed from both the IM (50 mas uniform random noise) and the FM, leaving only the correlated tides in. The runs were then repeated, all other things being equal, the following values were recorded: $\overline{\mu}=1.2104$, $\overline{\delta\mu}=0.1006$ with a variation with respect to the complete model case ($\overline{\mu}=1.2073$, $\overline{\delta\mu}=0.1295$) of: $\Delta\overline{\mu}\simeq 0.3\%$, $\Delta\overline{\delta\mu}\simeq 2.9\%$. Conversely, if all the signals with strong correlation are removed from the simulated data and from the FM, leaving only the uncorrelated ones in, we obtain: $\overline{\mu}=1.1587$, $\overline{\delta\mu}=0.0186$ with $\Delta\overline{\mu}\simeq 4.8\%$, $\Delta\overline{\delta\mu}\simeq 11\%$. Note that the sum of both contributions for $\Delta\overline{\mu}$ yields exactly the overall value $\Delta\overline{\mu}=5.2\%$ as obtained in the previous analysis (cfr. Fig.2 for $T_{obs}=4$ years). This highlights the importance of certain long period signals in affecting the recovery of LT and justify the choice of treating them simultaneously as it was done in obtaining Fig.2. Moreover, it clearly indicates that the efforts of the scientific community should be focused on the improvement of the knowledge of these tidal constituents. It has been tested that for $T_{obs}=8$ years all such "geometrical" correlations among the LT and the harmonics nearly disappear, as it would be intuitively expected.

5 The effect of the very long periodic harmonics

In this Section we shall deal with those signals whose periods are longer than 4 years which could corrupt the recovery of LT resembling superimposed trends if their periods are considerably longer than the adopted time series length.

We have shown the existence of a very long periodic ocean tidal perturbation acting upon the perigee of LAGEOS II. It is the K_1 $l=3$ $p=1$ $q=-1$ constituent with period $P=1851.9$

days (5.07 years) and nominal amplitudes of -1136 mas (Iorio 2000). In (Lucchesi 1998), for the effect of the direct solar radiation pressure on the perigee of LAGEOS II, it has been explicitly calculated, by neglecting the effects of the eclipses, a signal SRP(4241) with $P = 4241$ days (11.6 years) and nominal amplitude of 6400 mas. The mismodeling on these harmonics, both of the form $\delta A \sin\left(\frac{2\pi}{P}t + \phi\right)$, amounts to 64.5 mas for the tidal constituent, as shown in Section 1, and to 32 mas for SRP(4241), according to (Lucchesi 1998).

About the actual presence of such semisecular harmonics in the spectrum of the real combined residuals, it must be pointed out that, over $T_{obs} = 3.1$ years (Ciufolini et al. 1997), it is not possible that so low frequencies could be resolved by Fourier analysis. Indeed, according to (Godin 1972; Priestley 1981), when a signal is sampled over a finite time interval T_{obs} it induces a sampling also in the spectrum. The lowest frequency that can be resolved is:

$$f_{min} = \frac{1}{2T_{obs}}, \tag{9}$$

i.e. a harmonic must describe, at least, half a cycle over T_{obs} in order to be detected in the spectrum. f_{min} is called elementary frequency band and it also represents the minimum separation between two frequencies in order to be resolved. For our two signals we have $f(K_1) = 5.39 \cdot 10^{-4}$ cycles per day (cpd) and $f(SRP) = 2.35 \cdot 10^{-4}$ cpd; over 3.1 years $f_{min} = 4.41 \cdot 10^{-4}$ cpd. This means that the two harmonics neither can be resolved as distinct nor they can be found in the spectrum at all. In order to resolve them we should wait for $T_{obs} = 4.5$ years which corresponds to their separation $\Delta f = 3.04 \cdot 10^{-4}$ cpd, according to eq.(9). In view of the potentially large aliasing effect of these two harmonics on the LT it was decided to include both K_1 $l = 3$ $p = 1$ and SRP(4241) in the simulated residual curve at the level of mismodeling claimed before.

In order to obtain an upper bound of their contribution to the systematic uncertainty on μ_{LT} we proceeded as follows. First, we calculated the temporal average of the perturbations induced on the combined residuals by the two harmonics over different T_{obs} according to:

$$I = \frac{1}{T_{obs}} \int_0^{T_{obs}} k_2 \delta A \sin\left(\frac{2\pi}{P}t + \phi\right) dt = k_2 \frac{\delta A}{\tau}(\cos\phi + \sin\phi \sin\tau - \cos\phi\cos\tau), \tag{10}$$

with $\tau = 2\pi\frac{T_{obs}}{P}$. The results are shown in Fig.4 and Fig.5.

It can be seen that, for given T_{obs}, the averages are periodic functions of the initial phases ϕ with period 2π. Since the gravitomagnetic trend is positive and growing in time, we shall

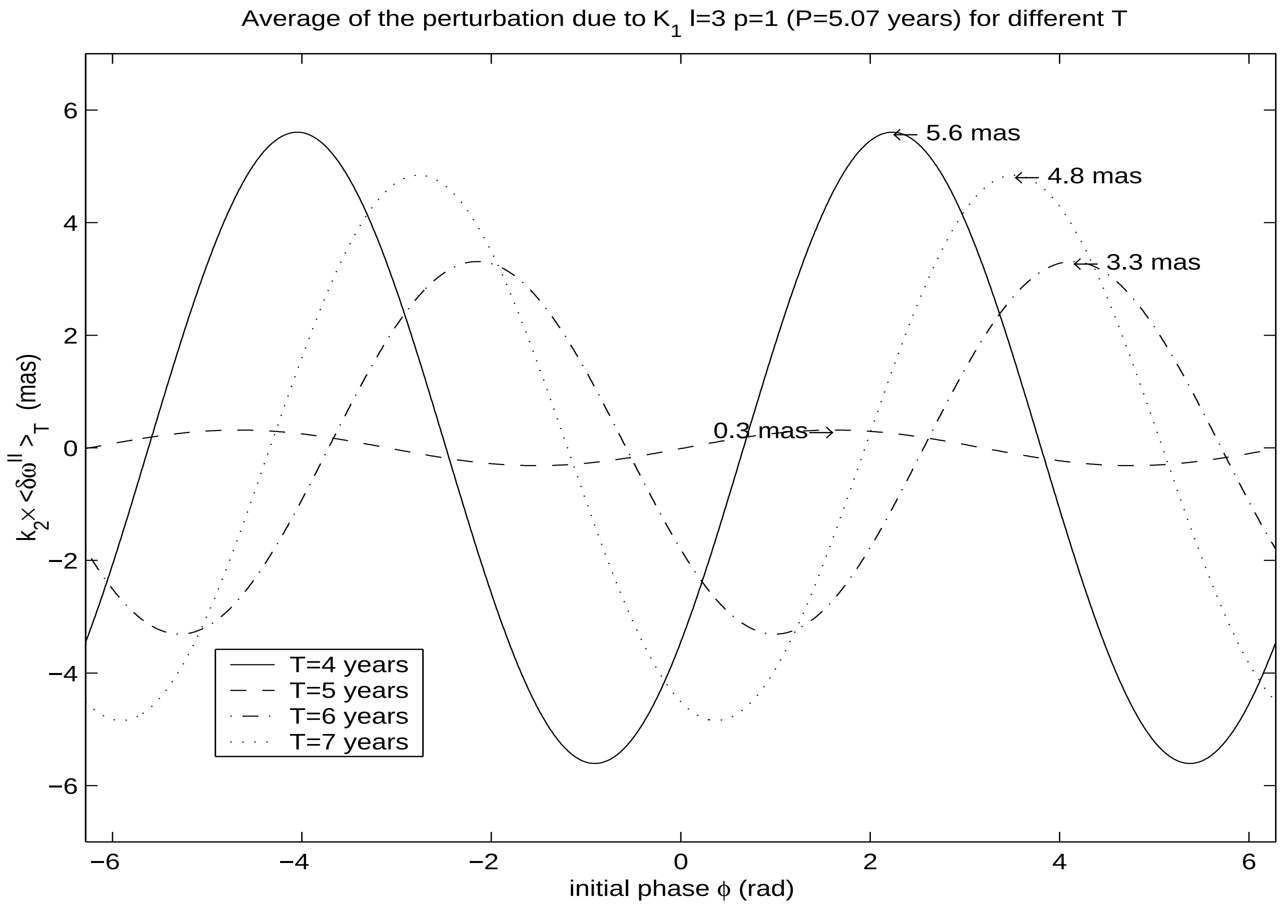

Figure 4: Average $k_2 < \delta\omega^{II} >_T$ over different T_{obs} of the perturbation induced on the combined residuals by the mismodeled harmonic K_1 $l = 3$ $p = 1$. In general, it depends on the initial phase ϕ. It has been calculated a mismodeling of 64.5 mas from a nominal amplitude of 1136 mas. It is mainly due to the C^{+}_{31f} ocean tidal coefficient and the load Love number $k^{'}_{3}$.

consider only the positive values of the temporal averages corresponding to those portions of sinusoid which are themselves positive. Moreover, note that, in this case, one should consider if the perturbation' s arc is rising or falling over the considered T_{obs}: indeed, even though the corresponding averages could be equal, it is only in the first case that the sinusoid has an aliasing effect on the LT trend. The values of ϕ which maximize the averages were found numerically and for such values the maxima of the averages were calculated. Subsequently, these results in mas were compared to the amount of the predicted gravitomagnetic shift accumulated over the chosen T_{obs} by the combined residuals $y_{LT} = 60.2$ (mas/y)$\times T_{obs}$ (y). The results are shown in Tab.3 and Tab.4 and, as previously outlined, represent a pessimistic estimate.

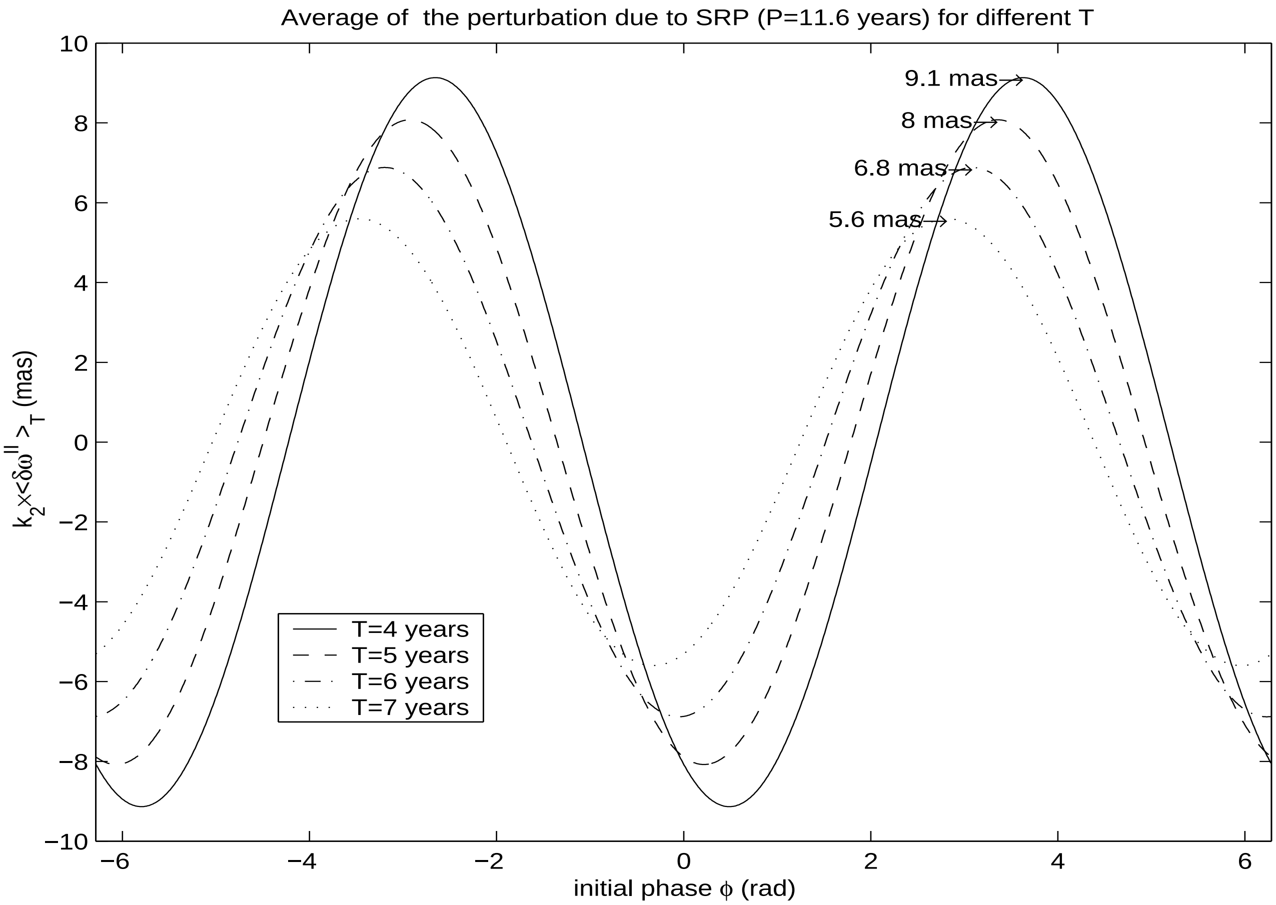

Figure 5: Average $k_2 < \delta\omega^{II} >_T$ over different T_{obs} of the perturbation induced on the combined residuals by the mismodeled harmonic SRP(4241). In general, it depends on the initial phase ϕ. It has been assumed a 0.5% level of mismodeling mainly due to the reflectivity coefficient C_R of LAGEOS II. The effects of the eclipses have been neglected.

Averaged mismodeled K_1 $l=3$ $p=1$			
T_{obs} (years)	$P(K_1\ l=3\ p=1) = 5.07$ years		
	max $k_2 < \delta\omega^{II} >$ (mas)	y_{LT} (mas)	max $k_2 < \delta\omega^{II} > /y_{LT}$ (%)
4	5.6	240.8	2.3
5	0.3	301	0.09
6	3.3	361.2	0.9
7	4.8	421.4	1.1

Table 3: Effect of the averaged mismodeled harmonic K_1 $l=3$ $p=1$ on the Lense-Thirring trend for different T_{obs}. In order to obtain upper limits the maximum value for the average of the tidal constituent has been taken, while for the gravitomagnetic effect it has been simply taken the value $\dot{y}_{LT} \times T_{obs}$.

Averaged mismodeled $SRP(4241)$			
T_{obs} (years)	$P(SRP) = 11.6$ years		
	max $k_2 < \delta\omega^{II} >$ (mas)	y_{LT} (mas)	max $k_2 < \delta\omega^{II} > /y_{LT}$ (%)
4	9.1	240.8	3.7
5	8	301	2.6
6	6.8	361.2	1.8
7	5.6	421.4	1.3

Table 4: Effect of the averaged mismodeled harmonic $SRP(4241)$ on the Lense-Thirring trend for different T_{obs}. In order to obtain upper limits the maximum value for the average of the radiative harmonic has been taken, while for the gravitomagnetic effect it has been simply taken the value $\dot{y}_{LT} \times T_{obs}$.

It is interesting to note that an analysis over $T_{obs} = 5$ years, that should not be much more demanding than the already performed works, could cancel out the effect of the K_1 $l = 3$ $p = 1$ tide; in this scenario the effect of SRP(4241) should amount, at most to 2.6% of the LT effect. Note also that for $T_{obs} = 7$ years, a time span sufficient for the LT to emerge on the background of most of the other tidal perturbations, the estimates of Fig.2 are compatible with those of Tab.3 and Tab.4 which predict an upper contribution of 1.1% from the K_1 $l = 3$ $p = 1$ and 1.3% from SRP(4241) on the LT parameter μ.

An approach, similar to that used in (Vespe 1999) in order to assess the influence of the eclipses and Earth penumbra effects on the perigee of LAGEOS II has been applied also to our case. It consists in fitting with a straight line only the mismodeled perturbation to be considered and , subsequently, comparing the slope of such fits to that due to gravitomagnetism which is equal to 1 in units of 60.2 mas/y. We have applied this method to K_1 $l = 3$ $p = 1$ and SRP(4241) with the already cited mismodeled amplitudes and by varying randomly the initial phases ϕ within $[-2\pi,\ 2\pi]$. The mean value of the fit' s slope, in units of 60.2 mas/y, over 1500 runs is very close to zero. This agrees with Fig.4 and Fig.5 which tell us that the temporal averages of K_1 $l = 3$ $p = 1$ and SRP(4241) are periodic functions of ϕ with period of 2π and, consequently, have zero mean value. Concerning the upper limits of $\Delta\mu$ derived from these runs, they agree with those released in Tab.3 and Tab.4 up to 1-2 %.

The method of temporal averages can be successfully applied also for the $l = 2$ $m = 0$ 18.6-year tide. It is a very long period zonal tidal perturbation which could potentially reveal itself as the most dangerous in aliasing the results for μ since its nominal amplitude is very large and its period is much longer than the T_{obs} which could be adopted for real analyses. Ciufolini

(1996) claims that the combined residuals have the merit of cancelling out all the static and dynamical geopotential' s contributions of degree $l = 2, 4$ and order $m = 0$, so that the 18.6-year tide would not create problems. This topic was quantitatively addressed in a preliminary way in (Iorio 2000). Indeed, in order to make comparisons with other works, we have simply calculated for $T_{obs} = 1$ year the combined residuals with only the mismodeled amplitudes of the 18.6-year tidal perturbations on the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II. This means that the dynamical pattern over the time span of such important tidal perturbation was not investigated. We did this by calculating the average over different T_{obs}. The results are in Fig.6 which shows clearly that the the 18.6-year tide does not affect at all the estimation of μ if we adopt as observable the combined residuals proposed by Ciufolini. Indeed, for $T_{obs} = 4$ years, the average effect will reach, at most, 0.08% of the gravitomagnetic shift over the same time span.

This feature of the 18.6-year tide is confirmed also by fitting with a straight line only the sinusoid of this perturbation on the combined residual: the adjusted slope amounts, at most, to less than 1% of the gravitomagnetic effect for different T_{obs}.

6 Conclusions

In this paper we have evaluated quantitatively the effects of mismodeling the orbital perturbations due to the tesseral and sectorial ocean and solid Earth tides on the combined nodes and perigee residuals of LAGEOS and LAGEOS II as proposed in (Ciufolini 1996) in order to detect the Lense-Thirring effect.

On one hand, by simulating the real residual curve in order to reproduce as closely as possible the results obtained for the $\Delta t = 15$ days $T_{obs} = 4$ years scenario published in (Ciufolini et al. 1998) it has been possible to refine and detail the estimates for it. On the other hand, this procedure also extends them to longer observational periods in view of new, more sophisticated analyses to be completed in the near future based on real data analyzed with the orbit determination software GEODYN II in collaboration with the teams from the Joint Center for Earth Systems Technology at NASA Goddard Space Flight Center and at the University of Rome La Sapienza. Since such numerical analyses are very demanding in terms of both time

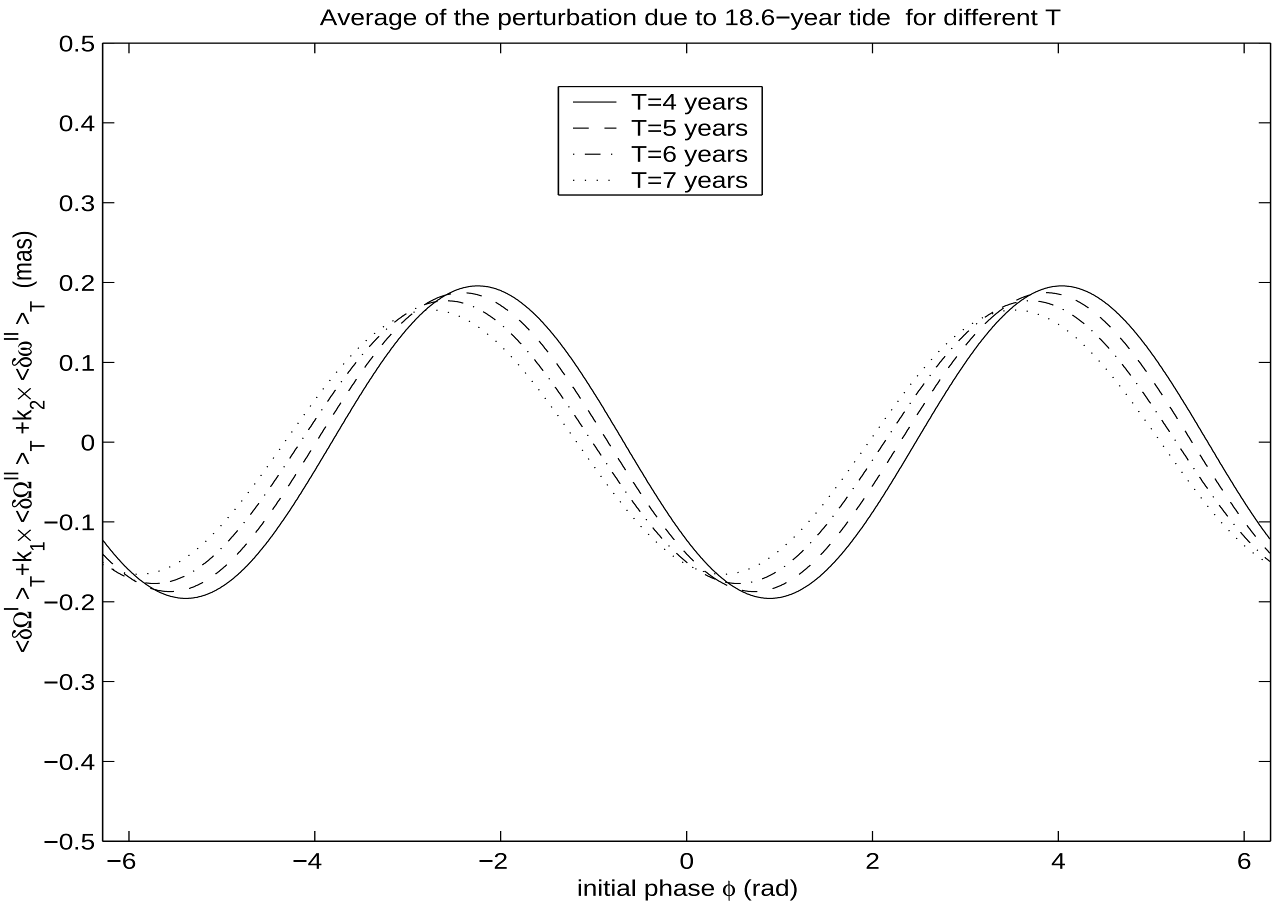

Figure 6: Average $< \delta\Omega^{I} >_T + k_1 < \delta\Omega^{II} >_T + k_2 < \delta\omega^{II} >_T$ over different T_{obs} of the perturbation induced on the combined residuals by the mismodeled 18.6-year tide. In general, it depends on the initial phase ϕ. It has been assumed a 1.5% level of mismodeling on the Love number k_2 mainly due to the anelasticity of the Earth' s mantle behaviour.

employed and results analysis burden, it should be very useful to have a priori estimates which could better direct the work. This could be done, e.g., by identifying which tidal constituents μ_{LT} is more or less sensitive to in order to seek improved dynamical models for use in GEODYN.

As far as the perturbations generated by the solid Earth tides, the high level of accuracy with which they are known has yielded a contribution to the systematic errors on μ_{LT} which falls well below 1%, so that they are of no concern at present.

Concerning the ocean tidal perturbations and the other long-period harmonics, for those whose periods are shorter than 4 years, the role played by T_{obs}, Δt and the noise has been investigated. It turned out that Δt has no discernible effect on the adjusted value of μ, while

T_{obs} is very important and so is the noise. The main results for these are summarized in Fig.2, which tells us that the entire set of long-period signals, if properly accounted for in building up the residuals, affect the recovery of the Lense-Thirring effect at a level not worse than $4\% - 5\%$ for $T_{obs} = 4$ years; the error contribution diminishes to about 2% after 7 years of observations.

We have also shown which tides are strongly anticorrelated and correlated with the gravitomagnetic trend over 4 years of observations. The experimental and theoretical efforts should concentrate on improving these constituents in particular. This geometrical correlation tends to diminish as T_{obs} grows. This can be intuitively recognized by noting that the longer T_{obs} is, the larger the number of cycles these periodic signals are sampled over and cleaner the way in which the secular Lense-Thirring trend emerges against the background "noise".

The ocean tide constituents K_1 $l = 3$ $p = 1$ and the solar radiation pressure harmonic SRP(4241) generate perturbations on the perigee of LAGEOS II with periods of 5.07 years and 11.6 years respectively, so that they act on the Lense-Thirring effect as biases and corrupt its determination with the related mismodeling: indeed they may resemble trends if T_{obs} is shorter than their periods. They were included in the simulated residual curve and their effect was evaluated in different ways with respect to the other tides. An upper bound was calculated for their action and it turns out that they contribute to the systematic uncertainty on the recovery of μ_{LT} at a level of less than 4% depending on T_{obs} and the initial phases ϕ. The results are summarized in Tab.3 and Tab.4. An observational period of 5 years, which seems to be a reasonable choice in terms of time scale and computational burden, allows to average out the effect of the K_1 $l = 3$ $p = 1$.

The strategy followed for the latter harmonics has been extended also to the $l = 2$ $m = 0$ zonal 18.6-year tide. Fig.6 confirms the claim in (Ciufolini 1996) that it does not affect the combined residuals.

In conclusion, the strategy presented here could be used as follows. Starting from a simulated residual curve based on the state of art of the real analyses performed until now, it provides helpful indications in order to improve the force models of the orbit determination software as far as tidal perturbations are concerned and to perform new analyses with real residuals. Moreover, when real data will be collected for a given scenario it will be possible to use them in our software in order to adapt the simulation procedure to the new situation; e.g. it is expected

that the noise level in the near future will diminish in view of improvements in laser ranging technology and modeling. Thus, we shall repeat our analyses for $\Delta\mu_{tides}$ when these new results become available.

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